

# $\bar{B}_s^0 \rightarrow (\pi^0 \eta^{(\prime)}, \eta^{(\prime)} \eta^{(\prime)})$ decays and the effects of next-to-leading order contributions in the perturbative QCD approach

Zhen-Jun Xiao<sup>1,2\*</sup> and Ya Li<sup>1</sup>, Dong-Ting Lin<sup>1</sup>, Ying-Ying Fan<sup>3</sup>, and Ai-Jun Ma<sup>1</sup>

1. *Department of Physics and Institute of Theoretical Physics,  
Nanjing Normal University, Nanjing, Jiangsu 210023, P.R. China*

2. *Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems,  
Nanjing Normal University, Nanjing 210023, P.R. China and*

3. *College of Physics and Electronic Engineering,  
Xinyang Normal University, Xinyang, Henan 464000, P.R. China*

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In this paper, we calculate the branching ratios and CP violating asymmetries of the five  $\bar{B}_s^0 \rightarrow (\pi^0 \eta^{(\prime)}, \eta^{(\prime)} \eta^{(\prime)})$  decays, by employing the perturbative QCD (pQCD) factorization approach and with the inclusion of all currently known next-to-leading order (NLO) contributions. We find that (a) the NLO contributions can provide about 100% enhancements to the LO pQCD predictions for the decay rates of  $\bar{B}_s^0 \rightarrow \eta \eta'$  and  $\eta' \eta'$  decays, but result in small changes to  $Br(\bar{B}_s \rightarrow \pi^0 \eta^{(\prime)})$  and  $Br(\bar{B}_s \rightarrow \eta \eta)$ ; (b) the newly known NLO twist-2 and twist-3 contributions to the relevant form factors can provide about 10% enhancements to the decay rates of the considered decays; (c) for  $\bar{B}_s \rightarrow \pi^0 \eta^{(\prime)}$  decays, their direct CP-violating asymmetries  $\mathcal{A}_f^{dir}$  could be enhanced significantly by the inclusion of the NLO contributions; and (d) the pQCD predictions for  $Br(\bar{B}_s \rightarrow \eta \eta^{(\prime)})$  and  $Br(\bar{B}_s \rightarrow \eta' \eta')$  can be as large as  $4 \times 10^{-5}$ , which may be measurable at LHCb or the forthcoming super-B experiments.

## I. INTRODUCTION

As is well-known, the studies for the mixing and decays of  $B_s$  meson play an important role in testing the standard model (SM) and in searching for the new physics beyond the SM [1, 2]. Some  $B_s$  meson decays, such as the leptonic decay  $B_s^0 \rightarrow \mu^+ \mu^-$  and the hadronic decays  $B_s^0 \rightarrow (J/\Psi \phi, \phi \phi, K \pi, K K, etc)$ , have been measured recently by the LHCb, ATLAS and CMS collaborations [3–5].

In a very recent paper [6], we studied the  $\bar{B}_s^0 \rightarrow (K \pi, K K)$  decays by employing the pQCD factorization approach with the inclusion of the NLO contributions [7–13] and found that the NLO contributions can interfere with the leading order (LO) part constructively or destructively for different decay modes, and can improve the agreement between the SM predictions and the measured values for the considered decay modes [6]. The charmless hadronic two-body decays of  $B_s$  meson, in fact, have been studied intensively by many authors by using rather different theoretical methods: such as the generalized factorization [14, 15], the QCD factorization (QCDF) approach [16–18] and the pQCD factorization approach at the LO or partial NLO level [8, 19–22]. In Refs.[7, 9, 10, 13], the authors proved that the NLO contributions can play a key role in

\* xiaozhenjun@njnu.edu.cn

understanding the very large  $Br(B \rightarrow K\eta')$  [9, 10], the so-called “ $K\pi$ -puzzle” [7, 13], and the newly observed branching ratios and CP violating asymmetries of  $B_s \rightarrow K^+\pi^-$  and  $B_s \rightarrow K^+K^-$  decays [3, 4, 6].

In this paper, we will calculate the branching ratios and CP violating asymmetries of the five  $B_s^0 \rightarrow (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$  decays by employing the pQCD approach. We focus on the studies for the effects of various NLO contributions to the five  $\bar{B}_s^0 \rightarrow (\pi^0\eta^{(\prime)}, \eta\eta, \eta\eta', \eta'\eta')$  decays, specifically those NLO twist-2 and twist-3 contributions to the form factors of  $B_s^0 \rightarrow \pi, \eta^{(\prime)}$  transitions [11, 12].

## II. DECAY AMPLITUDES AT LO AND NLO LEVEL

As usual, we treat the  $B_s$  meson as a heavy-light system and considered it at rest for simplicity. Using the light-cone coordinates, we define the emitted meson  $M_2$  moving along the direction of  $n = (1, 0, \mathbf{0}_T)$  and another meson  $M_3$  the direction of  $v = (0, 1, \mathbf{0}_T)$ , and we also use  $x_i$  to denote the momentum fraction of anti-quark in each meson:

$$P_{B_s} = \frac{m_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, \mathbf{0}_T), \quad P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, \mathbf{0}_T),$$

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (1)$$

After making the integration over  $k_1^-, k_2^-,$  and  $k_3^+$  we find the conceptual decay amplitude

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3$$

$$\cdot \text{Tr} \left[ C(t) \Phi_{B_s}(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right], \quad (2)$$

where  $b_i$  is the conjugate space coordinate of  $k_{iT}$ ,  $C(t)$  are the Wilson coefficients evaluated at the scale  $t$ , and  $\Phi_{B_s}$  and  $\Phi_{M_i}$  are wave functions of the  $B_s$  meson and the final state mesons. The Sudakov factor  $e^{-\tilde{S}(t)}$  and  $S_t(x_i)$  together suppress the soft dynamics effectively [23].

For the considered  $B_s$  decays with a quark level transition  $b \rightarrow q'$  with  $q' = (d, s)$ , the weak effective Hamiltonian  $H_{eff}$  can be written as [24]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qq'}^* \left\{ [C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu)] + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\}. \quad (3)$$

where  $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi constant, and  $V_{ij}$  is the Cabbibo-Kobayashi-Maskawa (CKM) matrix element,  $C_i(\mu)$  are the Wilson coefficients and  $O_i(\mu)$  are the four-fermion operators.

For  $B_s^0$  meson, we consider only the contribution of Lorentz structure

$$\Phi_{B_s} = \frac{1}{\sqrt{2N_c}} (\not{P}_{B_s} + m_{B_s}) \gamma_5 \phi_{B_s}(\mathbf{k}_1), \quad (4)$$

with the distribution amplitude widely used in literature [6, 8, 19, 20, 22]

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp \left[ -\frac{M_{B_s}^2 x^2}{2\omega_{B_s}^2} - \frac{1}{2}(\omega_{B_s} b)^2 \right], \quad (5)$$

where the parameter  $\omega_{B_s}$  is a free parameter and we take  $\omega_{B_s} = 0.50 \pm 0.05 \text{ GeV}$  for  $B_s$  meson. For fixed  $\omega_{B_s}$  and  $f_{B_s}$ , the normalization factor  $N_{B_s}$  can be determined through the normalization condition:  $\int \frac{d^4 k_1}{(2\pi)^4} \phi_{B_s}(\mathbf{k}_1) = f_{B_s}/(2\sqrt{6})$ .

For the light  $\pi$ ,  $K$ ,  $\eta_q$  and  $\eta_s$ , their wave functions are similar in form and can be defined as in Refs. [25–27]

$$\Phi(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_C}} \gamma_5 [\not{P} \phi^A(x) + m_0 \phi^P(x) + \zeta m_0 (\not{p} - 1) \phi_P^T(x)], \quad (6)$$

where  $P$  and  $m_0$  are the momentum and the chiral mass of the light mesons. When the momentum fraction of the quark (anti-quark) of the meson is set to be  $x$ , the parameter  $\zeta$  should be chosen as  $+1$  ( $-1$ ). The expressions of the relevant twist-2 ( $\phi^A(x)$ ) and twist-3 ( $\phi^{P,T}(x)$ ) distribution amplitudes of the mesons  $M = (\pi, K, \eta_q, \eta_s)$  and the relevant chiral masses can be found easily in Refs.[6, 10]. The relevant Gegenbauer moments  $a_i$  have been chosen as in Ref. [22]:

$$a_1^{\pi, \eta_q, \eta_s} = 0, \quad a_2^{\pi, \eta_q, \eta_s} = 0.44 \pm 0.22. \quad (7)$$

The values of other parameters are  $\eta_3 = 0.015$  and  $\omega = -3.0$ .

For the  $\eta$ - $\eta'$  system, we use the traditional quark-flavor mixing scheme: the physical states  $\eta$  and  $\eta'$  are related to the flavor states  $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$  and  $\eta_s = s\bar{s}$  through a single mixing angle  $\phi$ ,

$$\eta = \cos \phi \eta_q - \sin \phi \eta_s, \quad \eta' = \sin \phi \eta_q + \cos \phi \eta_s. \quad (8)$$

The relation between the decay constants  $(f_q, f_s)$  and  $(f_\eta^q, f_\eta^s, f_{\eta'}^q, f_{\eta'}^s)$ , as well as the chiral enhancement  $m_0^q$  and  $m_0^s$ , have been defined for example in Ref. [10]. The parameters  $f_q, f_s$  and  $\phi$  have been extracted from the data [28]:

$$f_q = (1.07 \pm 0.02) f_\pi, \quad f_s = (1.34 \pm 0.06) f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ, \quad (9)$$

with  $f_\pi = 130$  MeV.

### A. LO amplitudes

The five  $B_s^0 \rightarrow \pi^0 \eta^{(\prime)}, \eta \eta, \eta' \eta', \eta \eta'$  decays considered in this paper have been studied previously in Ref. [20, 22] by employing the pQCD factorization approach at the leading order. The decay amplitudes as presented in Ref. [20, 22] are confirmed by our recalculation. We here focus on the examination for the possible effects of all currently known NLO contributions to these five decay modes in the pQCD factorization approach. The relevant Feynman diagrams which may contribute to the considered  $B_s^0$  decays at the leading order are illustrated in Fig. 1. We firstly show the LO decay amplitudes.

For  $\bar{B}_s^0 \rightarrow \pi^0 \eta^{(\prime)}$  decays, the LO decay amplitudes are

$$\mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta) = \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_q) \cos \phi - \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_s) \sin \phi, \quad (10)$$

$$\mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta') = \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_q) \sin \phi + \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_s) \cos \phi, \quad (11)$$

with

$$\begin{aligned} \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_q) &= \xi_u (f_{B_s} F_{a\eta_q} a_2 + M_{a\eta_q} C_2) \\ &\quad - \frac{3}{2} \xi_t \left[ f_{B_s} F_{a\eta_q} (a_7 + a_9) + M_{a\eta_q} C_{10} + M_{a\eta_q}^{P_2} C_8 \right], \end{aligned} \quad (12)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}(\bar{B}_s^0 \rightarrow \pi^0 \eta_s) &= \xi_u (f_\pi F_{e\eta_s} a_2 + M_{e\eta_s} C_2) \\ &\quad - \frac{3}{2} \xi_t [f_\pi F_{e\eta_s} (a_9 - a_7) + M_{e\eta_s} (C_8 + C_{10})], \end{aligned} \quad (13)$$

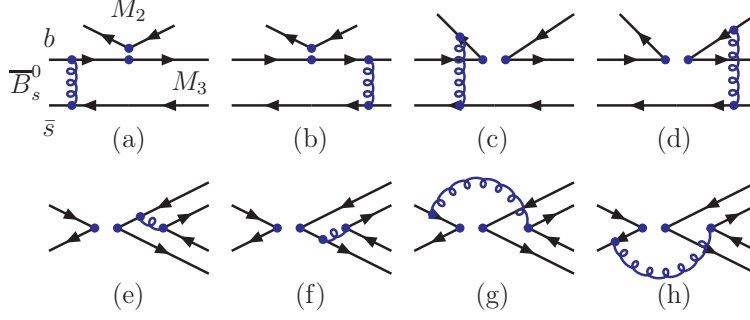


FIG. 1. Feynman diagrams which may contribute at leading order to  $B_s^0 \rightarrow (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$  decays.

where  $\xi_u = V_{ub}V_{us}^*$ ,  $\xi_t = V_{tb}V_{ts}^*$ , and  $a_i$  are the combinations of the Wilson coefficients  $C_i$  as defined for example in Ref.[10].

For  $\bar{B}_s^0 \rightarrow \eta\eta, \eta\eta', \eta'\eta'$  decays, the LO decay amplitudes are

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \rightarrow \eta\eta) = \cos^2 \phi \mathcal{A}(\eta_q\eta_q) - \sin(2\phi)\mathcal{A}(\eta_q\eta_s) + \sin^2 \phi \mathcal{A}(\eta_s\eta_s), \quad (14)$$

$$\mathcal{A}(\bar{B}_s^0 \rightarrow \eta\eta') = [\mathcal{A}(\eta_q\eta_q) - \mathcal{A}(\eta_s\eta_s)] \cos \phi \sin \phi + \cos(2\phi)\mathcal{A}(\eta_q\eta_s), \quad (15)$$

$$\sqrt{2}\mathcal{A}(\bar{B}_s^0 \rightarrow \eta'\eta') = \sin^2 \phi \mathcal{A}(\eta_q\eta_q) + \sin(2\phi)\mathcal{A}(\eta_q\eta_s) + \cos^2 \phi \mathcal{A}(\eta_s\eta_s), \quad (16)$$

with

$$\mathcal{A}(\bar{B}_s^0 \rightarrow \eta_q\eta_q) = \xi_u M_{a\eta_q} C_2 - \xi_t M_{a\eta_q} \left( 2C_4 + 2C_6 + \frac{1}{2}C_8 + \frac{1}{2}C_{10} \right), \quad (17)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\bar{B}_s^0 \rightarrow \eta_q\eta_s) &= \xi_u (f_q F_{e\eta_s} a_2 + M_{e\eta_s} C_2) - \xi_t \left[ f_q F_{e\eta_s} \left( 2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) \right. \\ &\quad \left. + M_{e\eta_s} \left( 2C_4 + 2C_6 + \frac{1}{2}C_8 + \frac{1}{2}C_{10} \right) \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{A}(\bar{B}_s^0 \rightarrow \eta_s\eta_s) &= -2\xi_t \left[ f_s F_{e\eta_s} \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \right. \\ &\quad \left. + (f_s F_{e\eta_s}^{P_2} + f_{B_s} F_{a\eta_s}^{P_2}) \left( a_6 - \frac{1}{2}a_8 \right) \right. \\ &\quad \left. + (M_{e\eta_s} + M_{a\eta_s}) \left( C_3 + C_4 + C_6 - \frac{1}{2}C_8 - \frac{1}{2}C_9 - \frac{1}{2}C_{10} \right) \right]. \end{aligned} \quad (19)$$

The individual decay amplitudes ( $F_{eM_3}, F_{eM_3}^{P_2}, \dots$ ) in Eqs. (12,13,17-19) are obtained by evaluating the Feynman diagrams in Fig. 1 analytically. Here ( $F_{eM_3}, F_{eM_3}^{P_2}$ ) and ( $M_{eM_3}, M_{eM_3}^{P_2}$ ) come from the evaluations of Figs.(1a,1b) and Figs.(1c,1d), respectively; while ( $F_{aM_3}, F_{aM_3}^{P_2}$ ) and ( $M_{aM_3}, M_{aM_3}^{P_2}$ ) are obtained by evaluating Figs.(1e,1f) and Figs.(1g,1h), respectively. One can find the expressions of all these decay amplitudes for example in Refs.[20, 22]. For the sake of the reader, we show  $F_{eM_3}$  and  $F_{eM_3}^{P_2}$  explicitly here:

$$\begin{aligned} F_{eM_3} &= 8\pi C_F M_{B_s}^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1, b_1) \\ &\quad \cdot \{ [(1+x_3)\phi_3^A(x_3) + r_3(1-2x_3)(\phi_3^P(x_3) + \phi_3^T(x_3))] \\ &\quad \cdot \alpha_s(t_e^1) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_e^1)] \\ &\quad + 2r_3\phi_3^P(x_3) \cdot \alpha_s(t_e^2) h_e(x_3, x_1, b_3, b_1) \exp[-S_{ab}(t_e^2)] \}, \end{aligned} \quad (20)$$

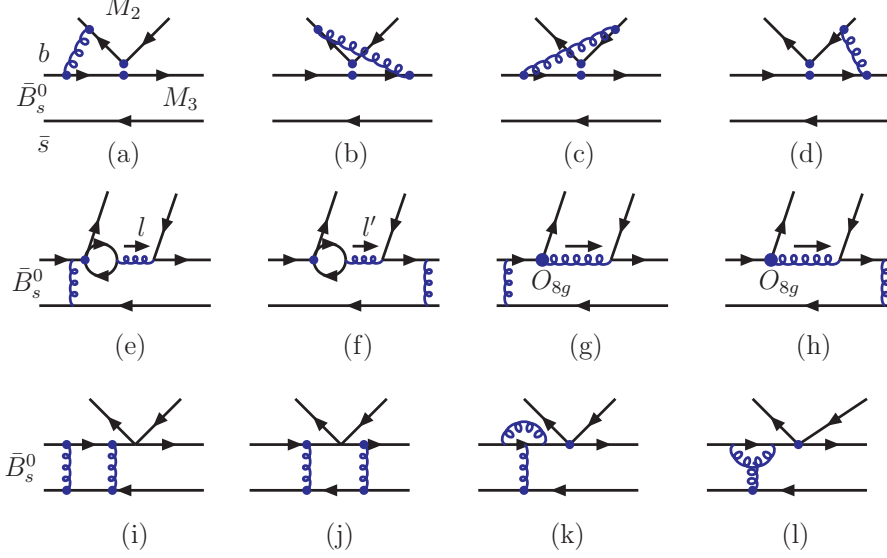


FIG. 2. Typical Feynman diagrams for NLO contributions: the vertex corrections (a-d); the quark-loops (e-f), the chromo-magnetic penguin contributions (g-h), and the NLO twist-2 and twist-3 contributions to  $B_s \rightarrow P$  transition form factors (i-l).

$$\begin{aligned}
 F_{eM_3}^{P_2} = & 16\pi C_F M_{B_s}^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1, b_1) r_2 \\
 & \cdot \left\{ [\phi_3^A(x_3) + r_3(2 + x_3)\phi_3^P(x_3) - r_3 x_3 \phi_3^T(x_3)] \right. \\
 & \cdot \alpha_s(t_e^1) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_e^1)] \\
 & \left. + 2r_3 \phi_3^P(x_3) \alpha_s(t_e^2) h_e(x_3, x_1, b_3, b_1) \exp[-S_{ab}(t_e^2)] \right\}, \quad (21)
 \end{aligned}$$

where  $C_F = 4/3$  is the color-factor,  $r_2 = m_0^{M_2}/M_{B_s}$  and  $r_3 = m_0^{M_3}/M_{B_s}$  with the chiral mass  $m_0$  for final state meson  $M_2$  and  $M_3$ . The explicit expressions of the hard energy scales ( $t_e^1, t_e^2$ ), the hard function  $h_e$  and the Sudakov factor  $\exp[-S(t)]$  can be found for example in Refs. [20, 22].

## B. NLO contributions

After many year's efforts, almost all NLO contributions in the pQCD approach become available now:

- (a) The NLO Wilson coefficients  $C_i(\mu)$  with  $\mu \approx m_b$  [24] and the strong coupling constant  $\alpha_s(\mu)$  at two-loop level.
- (b) The NLO vertex corrections (VC)[16], the NLO contributions from the quark-loops (QL) [7] or from the chromo-magnetic penguin (MP) operator  $O_{8g}$  [29]. The relevant Feynman diagrams are shown in Fig. 2(a)-2(h).
- (c) The NLO twist-2 and twist-3 contributions to the form factors of  $B \rightarrow P$  transitions (here P refers to the light pseudo-scalar mesons) [11, 12]. Based on the  $SU(3)$  flavor symmetry, we will extend directly the formulaes for NLO contributions to the form factors of  $B \rightarrow P$  transition as given in Refs. [11, 12] to the cases for  $B_s \rightarrow P$  transitions.

In this paper, we adopt the relevant formulae for all currently known NLO contributions directly from Refs. [6, 7, 10–12, 16, 29] without further discussion about the details. The still missing part of the NLO contributions in the pQCD approach is the calculation for the NLO corrections to the LO hard spectator and annihilation diagrams. But from the comparative studies for the LO and NLO contributions from different sources in Refs. [10, 13], we believe that those still unknown NLO contributions are most possibly the higher order corrections to the small LO quantities, and therefore can be neglected safely.

According to Refs. [7, 16], the vertex corrections can be absorbed into the re-definition of the Wilson coefficients by adding a vertex-function  $V_i(M)$  to them. The expressions of the vertex functions  $V_i(M)$  can be found easily in Refs. [7, 16]. The NLO "QL" and "MP" contributions are a kind of penguin correction with the insertion of the four quark operators and the chromo-magnetic operator  $O_{8g}$  respectively, as shown in Figs. 2(e,f) and 2(g,h). For the  $b \rightarrow s$  transition, the relevant effective Hamiltonian  $H_{eff}^{ql}$  and  $H_{eff}^{mp}$  can be written as the following form:

$$H_{eff}^{(ql)} = - \sum_{q=u,c,t} \sum_{q'} \frac{G_F}{\sqrt{2}} V_{qb}^* V_{qs} \frac{\alpha_s(\mu)}{2\pi} C^q(\mu, l^2) (\bar{b} \gamma_\rho (1 - \gamma_5) T^a s) (\bar{q}' \gamma^\rho T^a q'), \quad (22)$$

$$H_{eff}^{mp} = - \frac{G_F}{\sqrt{2}} \frac{g_s}{8\pi^2} m_b V_{tb}^* V_{ts} C_{8g}^{eff} \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a G_{\mu\nu}^a b_j, \quad (23)$$

where  $l^2$  is the invariant mass of the gluon which attaches the quark loops in Figs. 2(e,f), and the functions  $C^q(\mu, l^2)$  can be found in Ref. [7, 9]. The  $C_{8g}^{eff}$  in Eq. (23) is the effective Wilson coefficient with the definition of  $C_{8g}^{eff} = C_{8g} + C_5$  [7].

By analytical evaluations, we find that (a) the decay modes  $B_s^0 \rightarrow \pi^0 \eta^{(\prime)}, \eta_q \eta_q$  and  $\eta_q \eta_s$  do not receive the NLO contributions from the quark-loop and the magnetic-penguin diagrams; and (b) only the  $B_s^0 \rightarrow \eta_s \eta_s$  decay mode get the NLO contributions from the quark-loop diagrams and the  $O_{8g}$  operator:

$$\begin{aligned} \mathcal{M}_{\eta_s \eta_s}^{(ql)} = & -16m_{B_s}^4 \frac{C_F^2}{\sqrt{2N_c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 db_3 \phi_{B_s}(x_1) \left\{ [(1+x_3)\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^A(x_3) \right. \\ & + 2r_{\eta_s}\phi_{\eta_s}^P(x_2)\phi_{\eta_s}^A(x_3) + r_{\eta_s}(1-2x_3)\phi_{\eta_s}(x_2)(\phi_{\eta_s}^P(x_3) + \phi_{\eta_s}^T(x_3))] \\ & \cdot \alpha_s^2(t_a) \cdot h_e(x_1, x_3, b_1, b_3) \cdot \exp[-S_{ab}(t_a)] C^{(q)}(t_a, l^2) \\ & \left. + 2r_{\eta_s}\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^P(x_3) \cdot \alpha_s^2(t_b) \cdot h_e(x_3, x_1, b_3, b_1) \cdot \exp[-S_{ab}(t_b)] C^{(q)}(t_b, l^2) \right\}, \quad (24) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\eta_s \eta_s}^{(mp)} = & -32m_{B_s}^6 \frac{C_F^2}{\sqrt{2N_c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 db_3 \phi_{B_s}(x_1) \\ & \times \left\{ [(1-x_3) [2\phi_{\eta_s}^A(x_3) + r_{\eta_s}(3\phi_{\eta_s}^P(x_3) + \phi_{\eta_s}^T(x_3)) + r_{\eta_s}x_3(\phi_{\eta_s}^P(x_3) - \phi_{\eta_s}^T(x_3))] \phi_{\eta_s}^A(x_2) \right. \\ & - r_{\eta_s}x_2(1+x_3)(3\phi_{\eta_s}^P(x_2) - \phi_{\eta_s}^T(x_2))\phi_{\eta_s}^A(x_3)] \cdot \alpha_s^2(t_a) h_g(x_i, b_i) \cdot \exp[-S_{cd}(t_a)] C_{8g}^{eff}(t_a) \\ & \left. + 4r_{\eta_s}\phi_{\eta_s}^A(x_2)\phi_{\eta_s}^P(x_3) \cdot \alpha_s^2(t_b) \cdot h'_g(x_i, b_i) \cdot \exp[-S_{cd}(t_b)] C_{8g}^{eff}(t_b) \right\}, \quad (25) \end{aligned}$$

where the terms proportional to small quantity  $r_{\eta_s}^2$  are not shown explicitly. The expressions for the hard functions ( $h_e, h_g$ ), the functions  $C^{(q)}(t_a, l^2)$  and  $C^{(q)}(t_b, l^2)$ , the Sudakov functions  $S_{ab,cd}(t)$ , the hard scales  $t_{a,b}$  and the effective Wilson coefficients  $C_{8g}^{eff}(t)$ , can be found easily for example in Refs. [6, 7, 10].

The NLO twist-2 and twist-3 contributions to the form factors of  $B \rightarrow \pi$  transition have been calculated very recently in Refs. [11, 12]. Based on the  $SU(3)$  flavor symmetry, we extend the formulae of NLO contributions for  $B \rightarrow \pi$  transitions form factor as given in Refs. [11, 12] to the cases for  $B_s \rightarrow (\pi, \eta_q, \eta_s)$  transition form factors directly, after making appropriate replacements for some parameters. The NLO form factor  $f^+(q^2)$  for  $B_s \rightarrow \pi$  transition, for example, can be written as the form of

$$\begin{aligned}
f^+(q^2)|_{\text{NLO}} = & 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\
& \times \left\{ r_\pi [\phi_\pi^P(x_2) - \phi_\pi^T(x_2)] \cdot \alpha_s(t_1) \cdot e^{-S_{B_s\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \right. \\
& + \left[ (1 + x_2\eta) \left( 1 + F_{T2}^{(1)}(x_i, \mu, \mu_f, q^2) \right) \phi_\pi^A(x_2) + 2r_\pi \left( \frac{1}{\eta} - x_2 \right) \phi_\pi^T(x_2) - 2x_2 r_\pi \phi_\pi^P(x_2) \right] \\
& \quad \cdot \alpha_s(t_1) \cdot e^{-S_{B_s\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \\
& \left. + 2r_\pi \phi_\pi^P(x_2) \left( 1 + F_{T3}^{(1)}(x_i, \mu, \mu_f, q^2) \right) \cdot \alpha_s(t_2) \cdot e^{-S_{B_s\pi}(t_2)} \cdot S_t(x_2) \cdot h(x_2, x_1, b_2, b_1) \right\}, \quad (26)
\end{aligned}$$

where  $\eta = 1 - q^2/m_{B_s}^2$  with  $q^2 = (P_{B_s} - P_3)^2$  and  $P_3$  is the momentum of the meson  $M_3$  which absorbed the spectator light quark of the B meson,  $\mu$  ( $\mu_f$ ) is the renormalization (factorization) scale, the hard scale  $t_{1,2}$  are chosen as the largest scale of the propagators in the hard  $b$ -quark decay diagrams [11, 12], the function  $S_t(x_2)$  and the hard function  $h(x_i, b_j)$  can be found in Refs. [11, 12]. And finally the NLO factor  $F_{T2}^{(1)}(x_i, \mu, \mu_f, q^2)$  and  $F_{T3}^{(1)}(x_i, \mu, \mu_f, q^2)$  which describe the NLO twist-2 and twist-3 contribution to the form factor  $f^{+,0}(q^2)$  of the  $B_s \rightarrow \pi$  transition can be found in Refs. [6, 11, 12]. For  $B_s \rightarrow \pi$  transition, for example, these two factors can be written as:

$$\begin{aligned}
F_{T2}^{(1)} = & \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu^2}{m_{B_s}^2} - \left( \frac{13}{2} + \ln r_1 \right) \ln \frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16} \ln^2(x_1 x_2) + \frac{1}{8} \ln^2 x_1 \right. \\
& + \frac{1}{4} \ln x_1 \ln x_2 + \left( -\frac{1}{4} + 2 \ln r_1 + \frac{7}{8} \ln \eta \right) \ln x_1 + \left( -\frac{3}{2} + \frac{7}{8} \ln \eta \right) \ln x_2 \\
& \left. + \frac{15}{4} \ln \eta - \frac{7}{16} \ln^2 \eta + \frac{3}{2} \ln^2 r_1 - \ln r_1 + \frac{101\pi^2}{48} + \frac{219}{16} \right], \quad (27)
\end{aligned}$$

$$\begin{aligned}
F_{T3}^{(1)} = & \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu^2}{m_{B_s}^2} - \frac{1}{2} (6 + \ln r_1) \ln \frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16} \ln^2 x_1 - \frac{3}{8} \ln^2 x_2 \right. \\
& + \frac{9}{8} \ln x_1 \ln x_2 + \left( -\frac{29}{8} + \ln r_1 + \frac{15}{8} \ln \eta \right) \ln x_1 + \left( -\frac{25}{16} + \ln r_2 + \frac{9}{8} \ln \eta \right) \ln x_2 \\
& \left. + \frac{1}{2} \ln r_1 - \frac{1}{4} \ln^2 r_1 + \ln r_2 - \frac{9}{8} \ln \eta - \frac{1}{8} \ln^2 \eta + \frac{37\pi^2}{32} + \frac{91}{32} \right], \quad (28)
\end{aligned}$$

where  $r_i = m_{B_s}^2/\xi_i^2$  with the choice of  $\xi_1 = 25m_{B_s}$  and  $\xi_2 = m_{B_s}$ . For the considered  $B_s \rightarrow (\pi^0, \eta^{(\prime)}) \eta^{(\prime)}$  decays, the large recoil region corresponds to the energy fraction  $\eta \sim O(1)$ . We also set  $\mu = \mu_f = t$  in order to minimize the NLO contribution to the form factors [12, 32].



TABLE I. The pQCD predictions for the branching ratios ( in unit of  $10^{-6}$  ) of the considered five  $\bar{B}_s^0$  decays. As a comparison, we also list the theoretical predictions as given in Refs. [8, 16, 22], respectively.

Mode	LO	NLO-I	NLO	LO [22]	NLO-I[8]	QCDF [16]
$\bar{B}_s^0 \rightarrow \pi^0 \eta$	0.05	0.05	$0.06 \pm 0.03$	0.05	0.03	0.08
$\bar{B}_s^0 \rightarrow \pi^0 \eta'$	0.10	0.11	$0.13 \pm 0.06$	0.11	0.08	0.11
$\bar{B}_s^0 \rightarrow \eta \eta$	10.1	9.9	$10.6^{+3.8}_{-2.7}$	8.0	10.0	15.6
$\bar{B}_s^0 \rightarrow \eta \eta'$	27.5	38.4	$41.4^{+16.4}_{-12.0}$	21.0	34.9	54.0
$\bar{B}_s^0 \rightarrow \eta' \eta'$	20.5	37.7	$41.0^{+17.5}_{-13.4}$	14.0	25.2	41.7

### III. NUMERICAL RESULTS

In the numerical calculations the following input parameters (here the masses, decay constants and QCD scales are in unit of GeV) will be used [30, 31]:

$$\Lambda_{\overline{\text{MS}}}^{(5)} = 0.225, \quad f_{B_s} = 0.23, \quad f_\pi = 0.13, \quad m_{B_s} = 5.37, \quad m_\eta = 0.548, \quad m_{\eta'} = 0.958, \\ m_0^\pi = 1.4, \quad \tau_{B_s^0} = 1.497 \text{ ps}, \quad m_b = 4.8, \quad M_W = 80.41. \quad (29)$$

For the CKM matrix elements, we adopt the Wofenstein parametrization and use the following CKM parameters:  $\lambda = 0.2246$ ,  $A = 0.832$ ,  $\bar{\rho} = 0.130 \pm 0.018$  and  $\bar{\eta} = 0.350 \pm 0.013$ .

Taking  $B_s \rightarrow \pi$  transition as an example, we calculate and present the pQCD predictions for the form factors  $F_0^{\bar{B}_s^0 \rightarrow \pi}(0)$  at the LO and NLO level respectively:

$$F_0^{\bar{B}_s^0 \rightarrow \pi}(0) = \begin{cases} 0.22 \pm 0.05, & \text{LO,} \\ 0.24 \pm 0.05, & \text{NLO,} \end{cases} \quad (30)$$

where the error comes from the uncertainty of  $\omega_{B_s} = 0.50 \pm 0.05$  GeV,  $f_{B_s} = 0.23 \pm 0.02$  GeV and the Gegenbauer moments  $a_2^\pi = 0.44 \pm 0.22$ . Explicit calculations tell us that the NLO twist-2 enhancement to the full LO prediction is around 25%, but it is largely canceled by the negative NLO twist-3 contribution and finally lead to a small total enhancement (about 7%  $\sim$  9%) to the full LO prediction, as predicted in Ref. [12].

For the considered five  $\bar{B}_s^0$  decays, the CP-averaged branching ratios can be written in the following form:

$$\text{Br}(B_s^0 \rightarrow f) = \frac{G_F^2 \tau_{B_s}}{32\pi m_{B_s}} \frac{1}{2} \left[ |\mathcal{A}(\bar{B}_s^0 \rightarrow f)|^2 + |\mathcal{A}(B_s^0 \rightarrow \bar{f})|^2 \right], \quad (31)$$

where  $\tau_{B_s}$  is the lifetime of the  $B_s^0$  meson.

In Table I, we list the pQCD predictions for the CP-averaged branching ratios of the considered  $B_s^0$  decays. The label ‘‘NLO-I’’ means that all currently known NLO contributions are taken into account except for those to the form factors. As a comparison, we also show the central values of the LO pQCD predictions as given in Ref. [22], the partial NLO predictions in Ref. [8] and the QCDF predictions in Ref. [16] in last three columns of Table I. The main theoretical errors come from the uncertainties of the various input parameters: such as  $\omega_{B_s} = 0.50 \pm 0.05$ ,  $f_{B_s} = 0.23 \pm 0.02$  GeV and  $a_2^\pi = 0.44 \pm 0.22$ . The total errors of our pQCD predictions are obtained by adding the individual errors in quadrature.

From the numerical results as listed in Table I, one can observe the following points



- For  $\bar{B}_s^0 \rightarrow (\pi^0\eta, \pi^0\eta', \eta\eta)$  decays, the NLO enhancements to the full LO predictions are small in size: less than 30%. For  $\bar{B}_s^0 \rightarrow (\eta\eta', \eta'\eta')$  decays, however, the NLO enhancements can be as large as 100%. The branching ratios at the order of  $4 \times 10^{-5}$  should be measured at LHCb or super-B factory experiments.
- By comparing the numerical results as listed in the third (NLO-I) and fourth (NLO) column, one can see that the NLO contributions to the form factors along can provide  $\sim 10\%$  enhancement to the branching ratios.
- The pQCD predictions agree with the QCDF predictions within one standard deviation. The pQCD predictions given in some previous works [8, 22] are confirmed by our new calculations. Some differences between the central values are induced by the different choices of some input parameters, such as the Gegenbauer moments and the CKM matrix elements.
- The main theoretical errors are coming from the uncertainties of input parameters  $\omega_{B_s} = 0.50 \pm 0.05$ ,  $f_{B_s} = 0.23 \pm 0.02$  GeV and  $a_2^\pi = 0.44 \pm 0.22$ . The total theoretical error is in general around 30% to 50%.

Now we turn to the evaluations of the CP-violating asymmetries of the five considered decay modes. In the  $B_s$  system, we expect a much larger decay width difference:  $\Delta\Gamma_s/(2\Gamma_s) \sim -10\%$  [30]. Besides the direct CP violation  $\mathcal{A}_f^{dir}$ , the CP-violating asymmetry  $S_f$  and  $H_f$  are defined as usual [8, 22]

$$\mathcal{A}_f^{dir} = \frac{|\lambda|^2 - 1}{1 + |\lambda|^2}, \quad S_f = \frac{2\text{Im}[\lambda]}{1 + |\lambda|^2}, \quad H_f = \frac{2\text{Re}[\lambda]}{1 + |\lambda|^2}. \quad (32)$$

They satisfy the normalization relation  $|\mathcal{A}_f|^2 + |S_f|^2 + |H_f|^2 = 1$ , while the parameter  $\lambda$  is of the form

$$\lambda = \eta_f e^{2i\epsilon} \frac{A(\bar{B}_s^0 \rightarrow f)}{A(B_s^0 \rightarrow \bar{f})}, \quad (33)$$

where  $\eta_f$  is  $+1(-1)$  for a CP-even(CP-odd) final state  $f$  and  $\epsilon = \arg[-V_{ts}V_{tb}^*]$  is very small in size.

The pQCD predictions for the direct CP asymmetries  $\mathcal{A}_f^{dir}$ , the mixing-induced CP asymmetries  $S_f$  and  $H_f$  of the considered decay modes are listed in Table II and Table III. In these tables, the label “LO” means the LO pQCD predictions, the label “+VC”, “+QL”, “+MP”, as well as “NLO” means that the contributions from the vertex corrections, the quark loops, the magnetic penguins, and all known NLO contributions are added to the LO results, respectively. As a comparison, the LO pQCD predictions as given in Ref. [22] and the QCDF predictions in Ref. [16] are also listed in Table II and III. The errors here are defined in the same way as for the branching ratios.

From the pQCD predictions for the CP violating asymmetries of the five considered  $\bar{B}_s$  decays as listed in the Table II and III, one can see the following points:

- For  $\bar{B}_s^0 \rightarrow (\eta\eta, \eta\eta', \eta'\eta')$  decays, the pQCD predictions for  $\mathcal{A}_f^{dir}$  and  $S_f$  are very small: less than 3% in magnitude. The NLO effects are in fact also negligibly small.
- For  $\bar{B}_s^0 \rightarrow (\pi^0\eta, \pi^0\eta')$  decays, however, the NLO pQCD predictions for  $\mathcal{A}_f^{dir}$  can be as large as 40% – 52%. The NLO contributions can provide large enhancements to the LO pQCD predictions for  $\mathcal{A}_f^{dir}$ . Since the branching ratios of  $\bar{B}_s^0 \rightarrow (\pi^0\eta, \pi^0\eta')$  decays are at the  $10^{-8}$  level, unfortunately, there is no hope to observe their CP violation even at Super-B factory experiments.

TABLE II. The pQCD predictions for the direct CP asymmetries (in %) of the five  $\bar{B}_s^0$  decays. The meaning of the labels are described in the text.

Mode	LO	+ VC	+QL	+MP	NLO	pQCD[22]	QCDF[16]
$\bar{B}_s^0 \rightarrow \pi^0 \eta$	$-2.5^{+8.9}_{-7.8}$	39.8	—	—	$40.3^{+5.4}_{-7.5}$	$-0.4^{+0.3}_{-0.3}$	—
$\bar{B}_s^0 \rightarrow \pi^0 \eta'$	$24.7^{+0.3}_{-1.0}$	52.7	—	—	$51.9^{+2.9}_{-3.3}$	$20.6^{+3.4}_{-2.9}$	$27.8^{+27.2}_{-28.8}$
$\bar{B}_s^0 \rightarrow \eta \eta$	$-0.2^{+0.3}_{-0.2}$	-2.2	1.7	-1.8	$-2.3^{+0.5}_{-0.4}$	$-0.6^{+0.6}_{-0.5}$	$-1.6^{+2.4}_{-2.4}$
$\bar{B}_s^0 \rightarrow \eta \eta'$	$-1.1 \pm 0.1$	-1.0	0.1	-0.1	$-0.2 \pm 0.2$	$-1.3^{+0.1}_{-0.2}$	$0.4^{+0.5}_{-0.4}$
$\bar{B}_s^0 \rightarrow \eta' \eta'$	$1.4 \pm 0.2$	1.5	2.7	2.8	$2.8 \pm 0.4$	$1.9^{+0.4}_{-0.5}$	$2.1^{+1.3}_{-1.4}$

TABLE III. The pQCD predictions for the mixing-induced CP asymmetries (in %)  $S_f$  (the first row) and  $H_f$  (the second row). The meaning of the labels are the same as in Table II.

Mode	LO	+ VC	+QL	+MP	NLO	pQCD [22]
$\bar{B}_s^0 \rightarrow \pi^0 \eta$	$13.7^{+6.6}_{-8.3}$	11.3	—	—	$8.0^{+1.8}_{-2.7}$	$17^{+18}_{-13}$
	$99.0^{+0.5}_{-1.4}$	91.0	—	—	$91.2^{+3.0}_{-2.4}$	$99 \pm 1$
$\bar{B}_s^0 \rightarrow \pi^0 \eta'$	$-22.2^{+10.0}_{-7.3}$	-24.9	—	—	$-24.9^{+9.5}_{-6.1}$	$-17^{+8}_{-9}$
	$94.3^{+2.0}_{-1.8}$	81.3	—	—	$81.8^{+0.5}_{-0.1}$	$96^{+2}_{-2}$
$\bar{B}_s^0 \rightarrow \eta \eta$	$-0.6^{+0.4}_{-0.3}$	2.7	-1.8	-2.2	$-2.2^{+0.6}_{-0.5}$	$3.0^{+1}_{-1}$
	100.0	99.9	100.0	100.0	99.9	100.0
$\bar{B}_s^0 \rightarrow \eta \eta'$	$-0.1 \pm 0.1$	-0.4	0.1	0.1	$0.1 \pm 0.2$	4.0
	100.0	100.0	100.0	100.0	100.0	100.0
$\bar{B}_s^0 \rightarrow \eta' \eta'$	$0.8 \pm 0.1$	1.8	2.0	2.5	$2.5^{+0.2}_{-0.4}$	$4.0^{+1.0}_{-1.0}$
	100.0	100.0	99.9	99.9	99.9	100.0

#### IV. SUMMARY

In short, we calculated the branching ratios and CP-violating asymmetries of the five  $\bar{B}_s^0 \rightarrow (\pi^0, \eta^{(\prime)})\eta^{(\prime)}$  decays by employing the pQCD factorization approach. All currently known NLO contributions, specifically those NLO twist-2 and twist-3 contributions to the relevant form factors, are taken into account. From our studies, we found the following results:

- For  $\bar{B}_s^0 \rightarrow (\eta \eta', \eta' \eta')$  decays, the NLO enhancements to their branching ratios can be as large as 100%. For other three decay modes, however, the NLO enhancements are less than 30%. The newly known NLO twist-2 and twist-3 contributions to the form factors along can provide  $\sim 10\%$  enhancements to the branching ratios.
- For the  $\bar{B}_s \rightarrow \pi^0 \eta^{(\prime)}$  decays, the LO pQCD predictions for  $\mathcal{A}_f^{dir}$  can be enhanced significantly by the inclusion of the NLO contributions. For other three decays, the NLO contributions are small in size.
- For  $\bar{B}_s \rightarrow (\eta \eta^{(\prime)}, \eta' \eta')$  decays, their branching ratios are at the order of  $4 \times 10^{-5}$ , which may be measurable at LHCb or super-B factory experiments.

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